Phase-Only Transmit Beam Broadening for Improved Radar Search Performance

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Abstract - **The operational value of applying transmit beam broadening in radar search is considered. Test cases are established to evaluate search radar performance with beam broadening and without beam broadening, and also for different beam broadening arrangements. The test cases are evaluated according to search occupancy and probability of detection performance. The advantage of beam broadening for a horizon search fence is demonstrated; this approach extends to generic search fence design, and is particularly advantageous for broadening factors of 3:1 or higher.**

I. Introduction

This paper describes applications and design considerations for achieving objective energy distributions in a phased array transmit pattern using element-level phase as the independent variable. This approach is commonly referred to as phase-only beam spoiling when the broadening factor is small, or phase-only pattern synthesis (POPS) when the beam shape is changed more significantly. From a systems design and signal processing point of view, the POPS concept has a significant benefit since it allows a quasi decoupling of the far field transmit pattern from the antenna shape and size. This decoupling allows for additional degrees of freedom for multifunction phased arrays, and has also been used in the commercial sector to generate dish antenna patterns in communications satellites for efficient illumination of land masses in the satellite television market [1].

The motivation in this paper for considering POPS techniques and their application is driven by the following three distinct trends in current radar developments: (1) the use of large multi-function antennas and radars in space constrained environments, such as found on a typical naval vessels; (2) the application of solid state, or active, phased array radars to address a certain subset of these multi-function roles; and (3) the use of digital beamforming on receive in phased array radars for flexible operation and robust performance. The convergence of the first two trends in a single radar system tend to make the use of transmit beam broadening in conjunction with multiple simultaneous receive beams an attractive, or even essential, feature, while the third tends to make the realization of these approaches practicable.

Multi-function phased array radars in such a space constrained environment are often larger than optimum in aperture size for a given search or surveillance function. Such a situation can occur because the radar aperture has been sized for another critical function, such as long search and precision tracking. In such an instance the large aperture size leads to a beam that is too narrow for efficient implementation of near range search. It is the goal of this paper to examine two potential solutions to this problem, along with some of the ramifications of these approaches for energy management and detection performance.

The first approach considered in this paper is broadening of the transmit beam to accommodate the desire for wider field of view, a process that has come to be known colloquially as "shotgunning." The second approach considered here is the transmition of a burst cluster of beams in succession across the desired angular region quickly enough that the last beam is transmitted before the return of the first beam is received. This process is referred to colloquially as "machine gunning." In both cases, receive operation involves processing the returns using many simultaneous receive beams [2].

Multiple beam processing on receive is a well known subject with an extensive literature, and one might hope that the complementary problem on transmit could be treated with identical methods. This is not the case, however, since solid state phased arrays generally have a transmit power amplifier at each antenna element that is designed to operate well into saturation. This design choice confers several significant benefits, including lower total array cost, maximum power conversion efficiency, and minimal calibration requirements; however, under the condition of saturated operation, amplifier gain control at the element is not possible. This situation leaves the phase shifter setting as the only element level variable available for pattern control, and no trivial linear formulation of the transmit pattern synthesis problem can be formulated.

Despite the non-linearity, it is possible to synthesize arbitrary spatially band-limited antenna patterns – including broadened patterns – using only phase control; however, the beam efficiency for these patterns, defined as the actual directivity as a fraction of ideal directivity, can be prohibitively low[1]. The empirical penalty incurred for small

[1] Roughly, contours of functions generated by convolving the natural antenna pattern with the indicator function of an arbitrary measurable set can be synthesized.

broadening factors is well known to the antenna practitioner, although some surprising recent studies have shown [4] that for broadening factors larger than 2.5 beamwidths that phase only pattern synthesis can result in efficiencies greater than unity. One of the synthesized patterns studied in this paper exhibits this enhanced efficiency. The 3dB contours for the pattern main beams are shown in Figure 1.

Digital beamforming (DBF) in phased array radars is a growing trend for multiple reasons, including digital adaptive methods such as cancellation of undesired signals, improvement in instantaneous dynamic range, and multiple simultaneous receive beams. In the idealized DBF system with a digital receiver at every element, it is possible to form an arbitrarily large number and arbitrarily steered set of simultaneous receive beams, given sufficient computing and data handling resources. Many practical DBF systems do not have a digital receiver at every element, but rather a smaller number of digital receivers with each serving a number of antenna elements combined as a subarray. In the resulting subarray-level DBF, an arbitrarily large number of beams may still be formed digitally, but the possible angular or spatial extent of the beams will be limited by the directivity pattern of the combined subarray of antenna elements. In either case (element-level or subarray-level DBF) it is possible to use the multiple simultaneous receive beams in combination with "shotgun" or "machine gun" transmit beams to improve search occupancy and frame time.

This paper will examine the relative merits of "shotgun" versus "machine gun" transmit operation, advantageous situations for the application of each, and more detailed examination of particular approaches and methods to the "shotgun" approach. In order to simplify the analysis, we only consider element-level DBF in this paper and further assume that each element has an isotropic receive pattern. This allows us to model the receive gain curve as independent of steer direction, removing receive pattern considerations from the probability-of-detection (P_d) comparison between transmit beams. In practice, any implementation of the approaches considered here would require inclusion of the element gain and DBF architecture for accurate analysis.

II. Transmit Pattern Synthesis

The two approaches for search coverage discussed above For the purpose of illustration of the principles of MSRB for search, three simple test cases will be examined in detail for a fixed, notional solid state phased array radar antenna. The antenna will consist of a circular aperture of 100 wavelengths diameter. The test cases will each have four (4) MSRB, with variation in the transmit operation as follows:

- Case 1: "Machine gun" (MG) of four sequential beams in a 2 x 2 cluster;
- Case 2: "Shotgun" of a transmit beam broadened by a factor of 2:1 in each plane (2x2 SG);
- Case 3: "Shotgun" of a transmit beam broadened by a factor of 4:1 in one plane and unaltered in the other plane (1x4 SG).

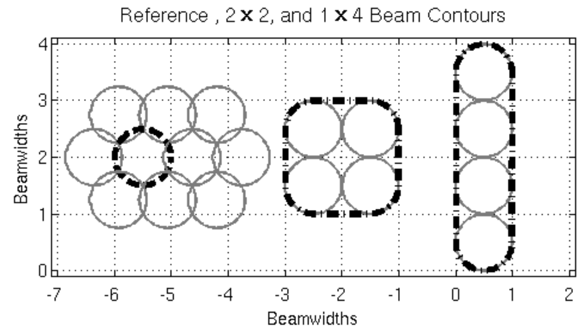


Figure 1 – Synthesized beam pattern definitions. Circular machine gun beams (right) are from a uniform element weighting. Contours relative to the nominal beams from broadened patterns are shown, 2:2 (middle) and 1:4 (left).

A. Transmit Pattern Synthesis Algorithm

The algorithmic approach used to generate the element level phase weightings used in this paper is described elsewhere [4], but is briefly summarized here for completeness.

A typical phase only pattern synthesis (POPS) problem will include broadening of the main beam along with simultaneous signal suppression in some sidelobe region. It is not assumed here that the array under consideration has periodic element layout, that the broadening region is elliptical, or that the signal suppression region is rectangular, as there are natural situations in which all of these conditions are undesirable. For example, the horizon maps into sine-space as an arc when the face of the array is tilted such that the horizon is off array broadside. Thus, a horizon notch is best described as a non-linear envelope around the horizon, not a rectangular keep-out region.

To develop a mathematical description of the POPS problem, define a region B in sine space, called the *broadening region*, within which the goal is to find element level phase settings that will cause the radiated antenna pattern to remain within 3dB of the maximum pattern value. We introduce the mathematical formulation of the POPS problem used for the results in this paper. Let

$$E(\mathbf{u};\varphi,a,\mathbf{x}) = \left| \sum_{k=1}^{N_e} a_k \, g_k(\mathbf{u}) e^{j\left(2\pi(\mathbf{x}_k - \bar{\mathbf{x}})^T \mathbf{u}/\lambda + \varphi_k\right)} \right|^2$$

denote the power density at a point **u** in sine space for given element phase weightings φ_k, amplitude weightings a_k, and element positions $\mathbf{x}_k$. The term $\bar{\mathbf{x}}$ denotes the mean of the element positions.

Problem 1: Take the amplitude weights to be fixed, and seek to select φ_k values to achieve the objective condition

$$M_B \le \min\left\{ E(\mathbf{u};\varphi,a) : \mathbf{u} \in B \right\}$$

for fixed M_B.

Problem 2: Define $E(\mathbf{u};\varphi,a)$ as above, and require that the condition, $\mathbf{C}^+$, that

$$M_B(\varphi) \le \min\left\{ E(\mathbf{u};\varphi,a) : \mathbf{u} \in B \right\}$$

is satisfied for the phase weighting φ, where

$$M_B(\phi) = 1/2 \max\left\{ E(\mathbf{u};\varphi,a) : \mathbf{u} \in B \right\}.$$

Assuming an elliptical region B in Problem 1, a natural choice of M_B is the 3dB value of the beam formed by a uniformly weighted aperture whose spatial dimensions give rise to the region B. Because of the larger aperture size, faster roll off is possible in the spoiled beam, leading to the possibility of packing more power into the main beam when solving the POPS problem. It can be shown [4] that this effect does not occur for small BF, but that the possibility is realized in both 1-D and 2-D broadening for larger BF; this fact has interesting consequences for the radar system design as discussed below.

We use the *beam efficiency* as a metric to determine how well a particular spoiled pattern performs against a nominal pattern. The beam efficiency is defined as the ratio of the energy in the sine space region enclosed by the 3dB contour of the beam under consideration to that of a uniformly weighted square aperture the same total radiated power as the beam under consideration.

B. Broadening Method

The broadening method used here is based on a homotopy from the known optimal beam with no broadening. Local optimality is maintained in an objective functional designed to generate the desired beam conditions, as described below.

The objective functional used here is based on decomposing the desired broadening region B into two disjoint regions; in the first, B^+, the power satisfies condition $\mathbf{C}^+$, while in the region B^- the power violates this condition. The objective function in this case attempts to maximize the power in the region B^+, while simultaneously penalizing the failure to achieve the desired power density in the region B^-. It is perhaps interesting to note that the second term in the formulation is required for beam broadening, as the uniform weighting seems to maximize the energy in B^+ independently of the geometry.

$$F(\varphi;B) = \int_{\mathbf{u}\in B^+} E(\mathbf{u};\varphi,a)\, d\mathbf{u} + \alpha \int_{\mathbf{u}\in B^-} \left(E(\mathbf{u};\varphi,a) - M_B(\phi)\right) d\mathbf{u}.$$

The homotopy method starts with a $B=B^{(0)}$ region that matches the main beam region of the uniform element weighting for $\alpha=0$. The value of α is gradually increased, with $\alpha \to \infty$ in the optimization limit. The algorithm is stopped by hand when it was determined that sufficient matching with the desired main beam region B^+ had been achieved.

C. Power Distribution Efficiency

Broadening the transmit beam can be more efficient in terms of the percentage of power that is concentrated into the desired beam area, as compared to conventional transmit beams. This is possible for broadened beams because energy that was formerly in sidelobes is incorporated into the broadened main beam. Less energy in the sidelobes means less wasted energy, and this can create an advantage for searching with broadened transmit beams. Taking this approach to its logical conclusion leads to the greatest possible efficiency of the so-called "ubiquitous radar" [3] (UR) in which none of the transmitted energy is wasted.

The quantitative benefit of the increased power distribution efficiency has been computed for various values of the beam broadening factor (BF). The BF is defined to be the ratio of the 3 dB width of the broadened beam to that of the original beam. In Figure 2 below, the efficiency as a function of BF is shown for a linear aperture distribution and a circular aperture distribution for values of BF from 1 to 15. The broadening is accomplished via phase-only pattern synthesis [4], since amplitude of the solid state phased array is normally not adjustable on transmit.

As the figure illustrates, there is a power distribution efficiency advantage for broadening factors of approximately 2.5 or greater, an advantage sometimes exceeding 2 dB for a 2-D array. For broadening factors of less than approximately 2.5, there is an efficiency penalty for the broadened beam. It will be shown that this may influence the choice of beam arrangement for a search operation.

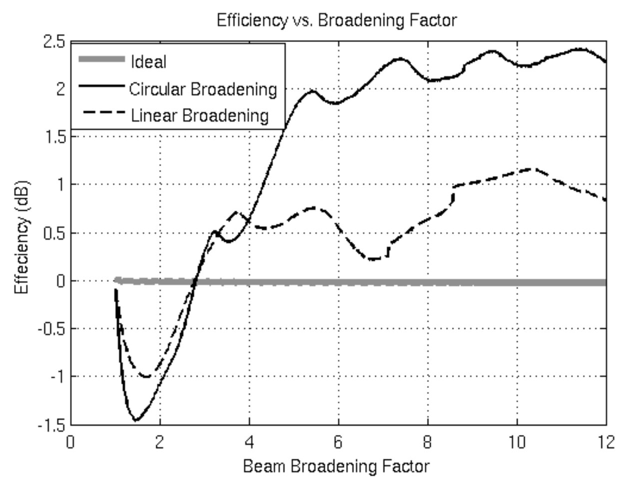


Figure 2 - Linear (1-D) and Circular (2-D) efficiencies as a function of broadening factor. The cases BF=1 corresponds to machine gunning, BF=2 (solid) corresponds to 2:2 broadening, and BF=4 (dashed) corresponds to 4:1 broadening.

In Figure 2, the power distribution efficiency of machine gun, 2×2, and 1×4 broadening are shown by the arrows. The 2×2 broadening corresponds to the idealized circular aperture broadening, while the 1×4 broadening can be idealized by linear aperture broadening. The two types of broadening result in roughly the same beam area, but the efficiency advantage of 1×4 vs. 2×2 broadening is more than 1.8 dB. There is also a small advantage of approximately 0.112 dB for 1×4 vs. conventional beamforming.

The antenna used to produce the 2x2 SG and 1x4 SG patterns is a circular array with triangular lattice of approximately 18,000 isotropic elements with nominally ½ wavelength spacing. In the horizon search problem considered below, the array was assumed to be tilted back at 15 degrees. As seen in Figure 3, Figure 4, and Figure 5 this arrangement creates an arc in sine space. This configuration was chosen to in order to evaluate performance of broadened beams in a non-separable array steered away from broadside. The beamwidth of this antenna is slightly larger than ½ degrees.

III. Transmit Beam Packing Efficiency

In the application of transmit pattern synthesis approaches to enabling multi-mission functionality, one has to carefully consider the comparison metric in order to understand the implications of a particular beam design on mission performance. The most natural metric to consider is the probability of detection, P_d, while an intuitive metric that is easy to compute is the beam packing efficiency (BPE). The BPE metric is discussed in this section, while P_d is considered in the rest of the paper.

Packing spoiled beams of spoiling factor BF may be more efficient than packing unspoiled beams because it covers a larger area than unspoiled beams. Using unity beamwidth, this concept is illustrated in Figure 1 where the area covered by a 1×4 spoiled beam is given as 3+π/4, the area covered by a 4 unspoiled beam is π, with a difference of 3-3π/4 ≅ 0.6438. Thus, the spoiled beam covers 0.6438/π or about 20 % more area than the 4 separate unspoiled beams. A larger ratio is obtained when the overlap between beams is considered.

That the spoiled beams pack together more efficiently can be seen by comparing Figure 5 with Figure 3. In Figure 5 20 1×4 spoiled beams are used to approximately cover the search area. If unspoiled beams are used to cover the same area in a triangular raster with overlap to ensure no voids, 115 beams are needed as illustrated in Figure 3. Discounting the left-right edge effects in the MG configuration, a slightly larger area is left unfilled in the spoiled patterns. The unfilled area in these two patterns is in fact the same, since every other top-bottom gap in the 1x4 SG pattern is moved to the center in the 2x2 SG pattern.

The BPE describes the efficiency in which the broadened beams may cover the search fence area relative to the MG configuration. This metric is defined as unity minus the ratio of the total radar resources required to cover the search fence for broadened to un-broadened beams (to within a small gap fraction, g). More precisely, let A_β and A_0 be the area of the 3dB contour of the broadened beam and reference beam, respectively, and N_β and N_0 be the number of beams required to cover the search area to within a gap g for the broadened and reference configurations. The gap condition is simply that $A_0 N_0 \geq |S| - g$. We also include the transmit efficiency (in linear units) to account for the average energy in the contour area. The BPE is defined as

$$BPE = \left(1 - \frac{1}{\alpha_\beta}\frac{N_\beta A_\beta}{N_0 A_0}\right) \times 100 \,. \qquad (1)$$

The efficiency of the 2x2 SG configuration used in this paper is -1.48dB, and the efficiency of the 1x4 SG configuration is 0.112dB. These are lower than the values obtained for the circular and linear arrays in Figure 2, but only cursory effort was made here to optimize the 2x2 SG and 1x4 SG patterns for efficiency. We expect the optimized patterns to have efficiencies that lie closer to the curves in Figure 2. Evaluation of the BPE for the two broadened configurations using (1) gives 18.3 for the 1x4 SG case and -17.9 for the 2x2 SG case. These BPE values serve as an indication that the 1x4 SG configuration is somewhat better than the reference configuration, while the 2x2 SG is less desirable.

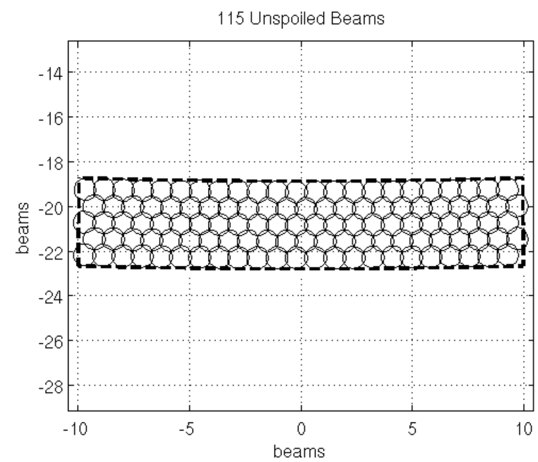


Figure 3 – 115 unspoiled MG beams packed in triangular raster with 20% overlap fraction.

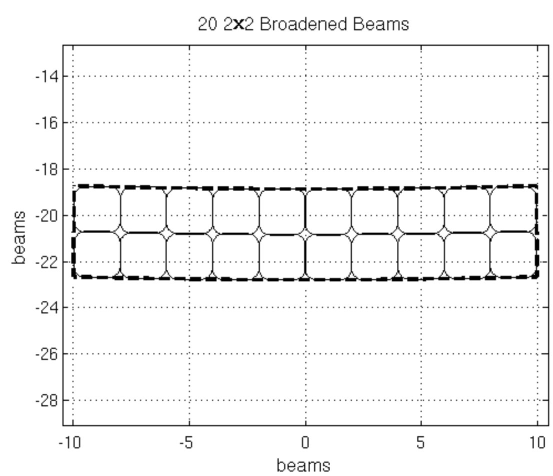


Figure 4 Two rows of 10 2x2 SG beams with 2:2 broadening factor.

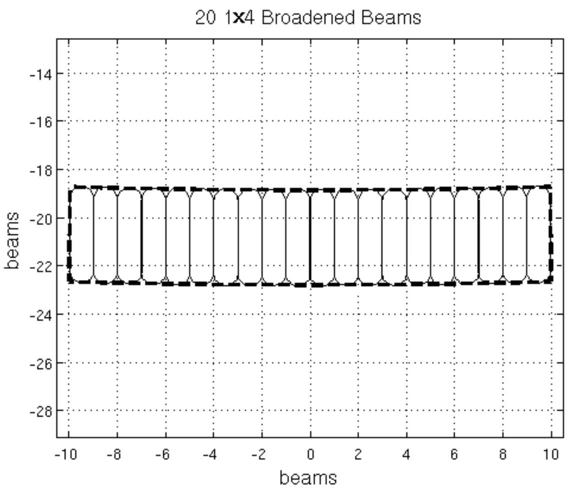


Figure 5 – One row of 20 4x1 SG beams with 4:1 broadening factor.

IV. Detection Probability Calculations

An analysis method is established in this section for comparison of the search performance between the POPS beam and machine gun configurations. A natural choice for comparison of the different methods is to examine the expected probability of detection over the entire search region. This comparison metric is normalized by constraining the radar resources to be fixed in some fixed reference time interval. One natural choice is to fix this time interval to be the time required for searching the surveillance region once with the reference configuration; this time will be called the search frame time and will be denoted by the variable *T*.

The search fence in each configuration is defined by the following parameters: the beam shape in sine space, $\mathsf{B}_\beta = \left\{ (u,v) \middle| G_\beta\left(u,v\middle|(0,0)\right) \geq \frac{1}{2}\mathrm{M}_\beta \right\}$, where $\mathrm{M}_\beta \equiv \max_{(u,v)} G_\beta\left(u,v\middle|(0,0)\right)$ denotes the maximum gain of the broadened beam; the beam center pointing directions of the search fence, $\mathsf{T} = \left\{ \left(u_1,v_1\right),\cdots,\left(u_{N_\beta},v_{N_\beta}\right) \right\}$; and the total search fence area , S. Indexing subscripts are used to denote the different configurations, with a zero subscript denoting the reference configuration characterized by the 20% overlap machine gun beam placement discussed above.

The constant search frame time requirement translates into the condition that the transmit time per beam in configuration β, t_β, is related to the number of beams used to cover the search fence, N_β, and the search frame time *T* through the formula $t_\beta N_\beta = T$. In particular, once a tiling configuration (T,B) is chosen for a given beam shape, the number of beams N_β is fixed, and the transmit time per beam in that configuration is determined by, $t_\beta = N_\beta^{-1} T = N_\beta^{-1} N_0 t_0$.

The signal-to-noise ratio (SNR) for a point in sine space (u,v) given nominal pointing direction (u_k, v_k), denoted by $SNR_\beta(u,v\,|\,u_k,v_k)$, can be related to the reference configuration beam center SNR for the target, denoted by SNR_0, through the formula

$$SNR_\beta(u,v\,|\,u_k,v_k) = SNR_0 \frac{N_0}{N_\beta G_0} G_\beta(u,v\,|\,u_k,v_k).$$

The following relationship between SNR, probability of false alarm (P_{fa}), and probability of detection (P_d) is used for calculation [5]:

$$P_d = P_{fa}^{\frac{1}{1+SNR}}. \tag{2}$$

Note that the probability of no detection is $1 - P_d$. For each point in sine space, we can compute the probability of detection given the beam pointing positions $\left\{ \left(u_1,v_1\right),\cdots,\left(u_{N_\beta},v_{N_\beta}\right) \right\}$ expressed as unity minus the product of the probabilities of no detection in any of the beams as follows:

$$\begin{aligned} P_d\left((u,v)\,\middle|\,\mathsf{T},\mathsf{B}\right) &= \\ &= 1 - \prod_{k=1}^{N_\beta}\left(1 - P_d\left(u,v\middle|(u_k,v_k)\right)\right) \qquad (3) \\ &= 1 - \prod_{k=1}^{N_\beta}\left(1 - P_{fa}^{\left(1+SNR_0\frac{N_0}{N_\beta G_0}G_\beta(u,v|u_k,v_k)\right)^{-1}}\right). \end{aligned}$$

This allows for the calculation of average probability of detection for a particular choice of transmit beam shape, tiling arrangement, and search region using the formula

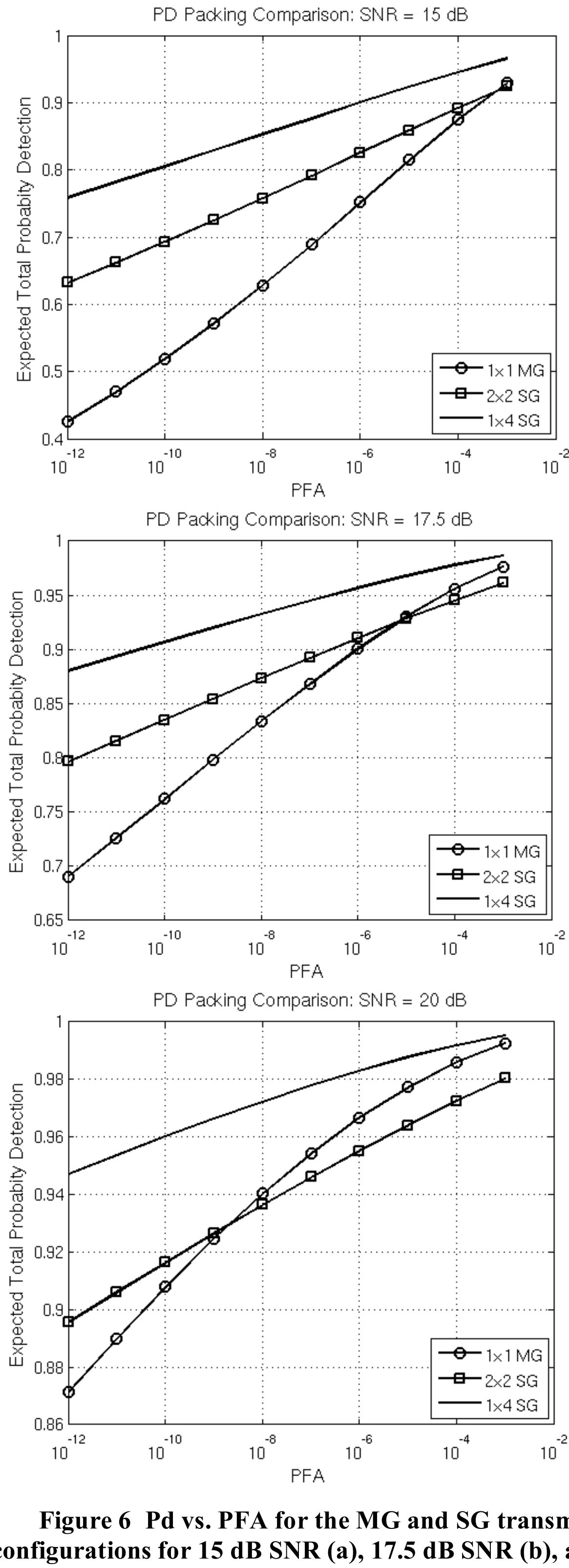
$$\bar{P}_d(\mathsf{T},\mathsf{B},\mathsf{S}) = \frac{1}{|\mathsf{S}|}\int_{(u,v)\in\mathsf{S}} P_d\left(u,v\,\middle|\,\mathsf{T},\mathsf{B}\right) du\,dv. \tag{4}$$

V. Analysis

The P_d performance was evaluated for the three cases in Figure 1 as a function of P_{fa} for three SNR values using the 18,000 element array. The results of this analysis are shown in Figure 6.

The 1x4 SG configuration had the highest detection probability across all P_{fa} and SNR values examined in this study. Intuitively, this is due to a greater-than-unity transmit efficiency combined with a more efficient geometrical packing than the reference configuration.

The performance of the 2x2 SG case is more complicated. At lower SNR and low P_{fa} this configuration out-performs the reference case. However, as the SNR is increased the reference case outperforms the 2x2 SG for all search configurations with a P_{fa} set above some crossover value. The crossover value is a function of SNR. Apparently, at low SNR the geometrical packing and the more uniformly flat main-beam response of the 2x2 SG configuration provides an advantage over the additional looks the MG configuration experiences, while at higher SNR the multiple looks and higher transmit efficiency of the MG configuration produces a higher PD.



(a)

(b)

(c)

Figure 6 Pd vs. PFA for the MG and SG transmit configurations for 15 dB SNR (a), 17.5 dB SNR (b), and 20 dB SNR (c). The 1:4 SG pattern outperforms 2:2 SG and MG in all cases; MG show relative improvement for higher PFA

VI. Conclusions

The use of phase only pattern synthesis for custom beam shaping in the horizon search problem was considered in this paper. A method for producing phase only weightings that result in broadened antenna patterns was discussed, and the method was used to generate two cases of broadened antenna patterns. The first pattern broadened the main beam equally in both azimuth and elevation (2x2 SG), while the second broadened the pattern in elevation only (1x4 SG).

Two metrics were discussed to evaluate search performance of these POPS patterns. The first is an intuitive metric that is simple to compute, while the second is more rigorous model for probability of detection over a single search frame that accounts for the possibility of detection from multiple beams. It was shown that the synthesis of a beam with large elevation spoiling could produce better probability of detection performance than a standard reference configuration of circular beams.

Although only one configuration of 1-D broadening was considered, it is expected that this performance advantage is generic for search regions of more than three reference beams in elevation. Such generic behavior is expected due to the enhanced beam efficiency for broadening factors greater than 3:1. This observation has significant implications for design of search strategies in large multifunction, multimission apertures.

VII. References

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